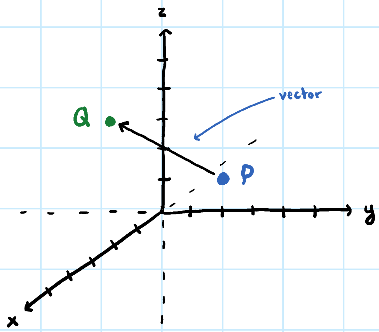
Vectors

Monday, May 8, 2023 8:54 AM

$P = (-1, 2, 1)$

$Q = (2, 0, 3)$



* P typically beginning point *

vector: given $P = (x_1, y_1, z_1)$ & $Q = (x_2, y_2, z_2)$ points, the vector $\vec{PQ}$ is denoted by (& defined by)

$$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

use ⟨ ⟩ for vector, not ( ) / ( ) for points

ex) $\vec{PQ} = \langle 2-(-1), 0-2, 3-1 \rangle$

$\vec{PQ} = \boxed{\langle 3, -2, 2 \rangle}$

* if flip values subtracted $\langle x_1 - x_2 \rangle$, then flip direction vector pointing / length same *

in fact, we also write $\langle v_1, v_2, v_3 \rangle$ a vector as:

$$v_1 \cdot \hat{\imath} + v_2 \cdot \hat{\jmath} + v_3 \cdot \hat{k}$$

(unit vectors i, j, & k)

ex) $\vec{PQ} = \langle 3, -2, 2 \rangle = \boxed{3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}$

length (magnitude): length of $\langle v_1, v_2, v_3 \rangle$ is given...

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

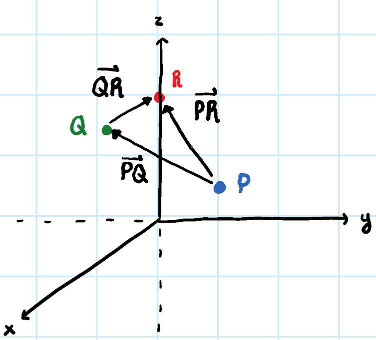
already includes subtraction of distance formula

ex) $\left.\begin{array}{l} P = (-1, 2, 1) \\ Q = (2, 0, 3) \end{array}\right\}$ $v = \vec{PQ} = \langle 3, -2, 2 \rangle$ then... $|v| = \sqrt{(3)^2 + (-2)^2 + 2^2} = \boxed{\sqrt{17}}$

$P = (-1, 2, 1)$

$Q = (2, 0, 3)$

$R = (0, 0, 4)$



two operations:

1) add vectors : $\langle v_1, v_2, v_3 \rangle + \langle w_1, w_2, w_3 \rangle = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$

ex) $\vec{PQ} + \vec{QR} = \langle \overset{v_1}{3}, \overset{v_2}{-2}, \overset{v_3}{2} \rangle + \langle \overset{w_1}{-2}, \overset{w_2}{0}, \overset{w_3}{1} \rangle$

$= \boxed{\langle 1, -2, 3 \rangle}$

check: $\vec{PR} = \langle 0 - -1, 0 - 2, 4 - 1 \rangle$

$\vec{PR} = \langle 1, -2, 3 \rangle$

2) multiply by constant (scalar multiplication) : $v = \langle v_1, v_2, v_3 \rangle$, c real # , then...

$$c \cdot v = \langle c \cdot v_1, c \cdot v_2, c \cdot v_3 \rangle$$

remark: same direction (arrow flips if $c < 0$), but length changes to

$$|c \cdot v| = |c| \cdot |v|$$

absolute value

length